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Method of wave equations exact solutions in studies of neutrinos and electrons interaction in dense matter

A I Studenikin

Department of Theoretical Physics, Moscow State University, 119992 Moscow, Russia

E-mail: studenik@srd.sinp.msu.ru

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Abstract
We present quite a powerful method in investigations of different phenomena that can appear when neutrinos and electrons propagate in background matter. This method implies use of exact solutions of modified Dirac equations that contain the correspondent effective potentials accounting for the matter influence on particles. For several particular cases the exact solutions of modified Dirac and Dirac–Pauli equations for a neutrino and an electron in the background environment of different composition are obtained (the case of magnetized matter is also considered). Neutrino reflection, trapping, neutrino pair creation and annihilation in matter and neutrino energy quantization in a rotating medium are discussed. The neutrino Green functions in matter are also derived. The two recently proposed mechanisms of electromagnetic radiation by a neutrino and an electron in matter (the spin light of neutrino and electron, SLν and SLe) are considered. A possibility to introduce an effective 'matter-induced Lorentz force' acting on a neutrino and an electron is discussed. A new mechanism of electromagnetic radiation that can be emitted by an electron moving in the neutrino background with nonzero gradient of density is predicted.

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1. Introduction

The problem of particles interactions under an external environment influence, provided by the presence of external electromagnetic fields or media, is one of the important issues of particle physics. In addition to possibility for better visualization of fundamental properties of particles and their interactions being imposed by influence of an external conditions, the interest to this problem is also stimulated by important applications to description of different processes in astrophysics and cosmology, where strong electromagnetic fields and dense matter may play an important role.

The aim of this paper is to present a rather powerful method in investigations of different phenomena that can appear when neutrinos are moving in the background matter [1, 2]. In addition, we also demonstrate how this method can be applied to electrons moving in background matter [3–7]. The developed new approach [4] establishes a basis for investigation of different phenomena which can arise when neutrinos and electrons move in dense media, including those peculiar for astrophysical and cosmological environments.

The method discussed is based on the use of the modified Dirac equations for the particles wavefunctions, in which the correspondent effective potentials accounting for matter influence on the particles are included. It is similar to the Furry representation [8] in quantum electrodynamics, widely used for the description of particles interactions in the presence of external electromagnetic fields. In this technique, the evolution operator $U_F(t_1, t_2)$, which determines the matrix element of the process, is represented in the usual form

$$U_F(t_1, t_2) = T\exp\left[-\mathrm{i}\int_{t_1}^{t_2} j_\mu(x)A^\mu\,\mathrm{d}x\right], \tag{1}$$

where $A_\mu(x)$ is the quantized part of the potential corresponding to the radiation field, which is accounted within the perturbation-series techniques. At the same time, the electron (a charged particle) current is represented in the form

$$j_\mu(x) = \frac{e}{2}[\overline{\Psi}_e\gamma_\mu, \Psi_e], \tag{2}$$

where Ψ_e are the exact solutions of the Dirac equation for the electron in the presence of external electromagnetic field given by the classical non-quantized potential $A_\mu^{\mathrm{ext}}(x)$,

$$\{\gamma^\mu\big(\mathrm{i}\partial_\mu - eA_\mu^{cl}(x)\big) - m_e\}\Psi_e(x) = 0. \tag{3}$$

Note that within this approach the interaction of charged particles with the external electromagnetic field is taken into account exactly while the radiation field is allowed for by perturbation-series expansion techniques. A detailed discussion of the use of this method can be found in [9]. Many processes with electrons under the influence of external electromagnetic fields were investigated using this method. In particular, this method was applied [10] for derivation of an electron dispersion relation in external electromagnetic fields as well as in studies of the problem of the electron anomalous magnetic moment in external fields (see [11] for a review).

In section 2.1, we derive the modified Dirac equation for the neutrino wavefunction in the presence of matter and find its exact solutions including the neutrino energy spectrum (section 2.2). On this basis we discuss the neutrino reflection, trapping and also neutrino pair annihilation and creation in matter (section 2.3). In section 2.4, we consider the modified Dirac equation for the case when neutrino propagates in rotating matter and find that its energy is quantized very much similar to the electron energy Landau quantization in a magnetic field. In section 2.5, we consider the Dirac–Pauli equation for a neutrino moving in matter. The correspondent neutrino energy spectrum, as well as the one of the modified Dirac equation, can be used for obtaining the correct values for the flavour and helicity neutrino energy difference in matter (section 2.6). In section 2.7, we use the modified Dirac–Pauli equation to get neutrino energy spectrum in magnetized and polarized matter. Section 2.8 is devoted to discussion of the modified Dirac equation for a Majorana neutrino in matter. Neutrino Green functions, for both Dirac and Majorana cases, are derived in section 3. In section 4, we apply the developed method of exact solutions of the quantum wave equation to the study of an electron moving in background matter and found exact solutions of the correspondent Dirac equation. In section 5, we illustrate how the obtained exact solutions can be used in studies of different processes in matter. As two examples, we discuss evaluation of quantum theory of the spin light of neutrino (SLν) and spin light of electron (SLe) in matter, the two recently discussed new mechanisms of electromagnetic radiation produced by a neutrino and an electron moving in matter. A possibility to introduce an effective 'matter-induced Lorentz force' acting on a neutrino and an electron is discussed in conclusions section 6. We also predict a new mechanism of electromagnetic radiation that can be emitted by an electron moving in the neutrino background with nonzero gradient of density. The proposed mechanism of the electromagnetic radiation can be important in physics of neutron stars, gamma-ray bursts and black holes.

2. Quantum equations for neutrino in matter

2.1. Modified Dirac equation for neutrino in matter

In [1] (see also [2, 3]) we derived the modified Dirac equation for neutrino wavefunction exactly accounting for the neutrino interaction with matter. Let us consider the case of matter composed of electrons, neutrons and protons, and also suppose that the neutrino interaction with background particles is given by the standard model supplied with the singlet right-handed neutrino. The corresponding addition to the neutrino effective interaction Lagrangian is given by

$$\Delta L_{\mathrm{eff}} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\right), \qquad f^{\mu} = \sqrt{2}G_{F}\sum_{f=e,p,n} j_{f}^{\mu}q_{f}^{(1)} + \lambda_{f}^{\mu}q_{f}^{(2)}, \tag{4}$$

where

$$q_{f}^{(1)} = \left(I_{3L}^{(f)} - 2Q^{(f)}\sin^{2}\theta_{W} + \delta_{ef}\right), \qquad q_{f}^{(2)} = -\left(I_{3L}^{(f)} + \delta_{ef}\right), \qquad \delta_{ef} = \begin{cases} 1 & \text{for } f = e, \\ 0 & \text{for } f = n, p. \end{cases} \tag{5}$$

Here $I_{3L}^{(f)}$ and $Q^{(f)}$ are, respectively, values of the isospin third components and the electric charges of matter particles ($f = e, n, p$). The corresponding currents j_{f}^{μ} and polarization vectors λ_{f}^{μ} are

$$j_{f}^{\mu} = (n_{f}, n_{f}\mathbf{v}_{f}), \qquad \lambda_{f}^{\mu} = \left(n_{f}(\boldsymbol{\zeta}_{f}\mathbf{v}_{f}), n_{f}\boldsymbol{\zeta}_{f}\sqrt{1-v_{f}^{2}} + \frac{n_{f}\mathbf{v}_{f}(\boldsymbol{\zeta}_{f}\mathbf{v}_{f})}{1+\sqrt{1-v_{f}^{2}}}\right), \tag{6}$$

where θ_{W} is the Weinberg angle. In the above formulae (6), n_{f}, $\mathbf{v}_{f}$ and $\boldsymbol{\zeta}_{f} (0 \leqslant |\boldsymbol{\zeta}_{f}|^{2} \leqslant 1)$ stand, respectively, for the invariant number densities, average speeds and polarization vectors of the matter components. Using the standard model Lagrangian with the extra term (4), we derive the modified Dirac equation for the neutrino wavefunction in matter [1],

$$\left\{\mathrm{i}\gamma_{\mu}\partial^{\mu} - \tfrac{1}{2}\gamma_{\mu}(1+\gamma_{5})f^{\mu} - m\right\}\Psi(x) = 0. \tag{7}$$

This is the most general form of the equation for the neutrino wavefunction in which the effective potential $V_{\mu} = \frac{1}{2}(1+\gamma_{5})f_{\mu}$ includes both the neutral and charged current interactions of neutrino with the background particles and which can also account for effects of matter motion and polarization. It should be mentioned that other modifications of the Dirac equation were previously used in [12–18] for studies of the neutrino dispersion relations, neutrino mass generation and neutrino oscillations in the presence of matter.

2.2. Neutrino quantum states in matter

In the further discussion below we consider the case when matter is composed of electrons and no electromagnetic field is present in the background. We also suppose that the matter is unpolarized, $\lambda^{\mu} = 0$. Therefore, the term describing the neutrino interaction with the matter is given by

$$f^{\mu} = \frac{\tilde{G}_F}{\sqrt{2}}(n, n\mathbf{v}), \tag{8}$$

where we use the notation $\tilde{G}_F = G_F(1 + 4\sin^2\theta_W)$.

For the stationary states of equation (7) we get [1]

$$\Psi(\mathbf{r}, t) = \mathrm{e}^{-\mathrm{i}(E_\varepsilon t - \mathbf{pr})} u(\mathbf{p}, E_\varepsilon), \tag{9}$$

where $u(\mathbf{p}, E_\varepsilon)$ is independent on the coordinates and time. Upon the condition that equation (7) has a non-trivial solution, we arrive to the energy spectrum of a neutrino moving in the background matter

$$E_\varepsilon = \varepsilon\eta\sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m, \tag{10}$$

where we use the notation

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m}, \tag{11}$$

and also introduce the value $\eta = \mathrm{sign}\left(1 - s\alpha\frac{m}{p}\right)$ in order to provide a proper behaviour of the wavefunction in the hypothetical massless case. The values $s = \pm 1$ specify the two neutrino helicity states, ν_+ and ν_-. In the relativistic limit the negative-helicity neutrino state is dominated by the left-handed chiral state ($\nu_- \approx \nu_L$), whereas the positive-helicity state is dominated by the right-handed chiral state ($\nu_+ \approx \nu_R$). The quantity $\varepsilon = \pm 1$ splits the solutions into the two branches that in the limit of the vanishing matter density, $\alpha \to 0$, reproduce the positive- and negative-frequency solutions, respectively. It is also important to note that the neutrino energy in the background matter depends on the state of the neutrino longitudinal polarization, i.e. in the relativistic case the left-handed and right-handed neutrinos with equal momenta have different energies.

We get the exact solution of the modified Dirac equation in the form [1]

$$\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r}, t) = \frac{\mathrm{e}^{-\mathrm{i}(E_\varepsilon t - \mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix} \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}}\sqrt{1 + s\frac{p_3}{p}} \\ s\sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}}\sqrt{1 - s\frac{p_3}{p}}\,\mathrm{e}^{\mathrm{i}\delta} \\ s\varepsilon\eta\sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}}\sqrt{1 + s\frac{p_3}{p}} \\ \varepsilon\eta\sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}}\sqrt{1 - s\frac{p_3}{p}}\,\mathrm{e}^{\mathrm{i}\delta} \end{pmatrix}, \tag{12}$$

where the energy E_ε is given by (10), L is the normalization length and $\delta = \arctan p_2/p_1$. In the limit of vanishing density of matter, when $\alpha \to 0$, the wavefunction (12) transforms to the vacuum solution of the Dirac equation.

Let us now consider in some detail properties of a neutrino energy spectrum (10) in the background matter that are very important for understanding of the mechanism of the neutrino spin light phenomena. For the fixed magnitude of the neutrino momentum p there are two values for the 'positive sign' ($\varepsilon = +1$) energies

$$E^{s=+1} = \sqrt{\mathbf{p}^2\left(1 - \alpha\frac{m}{p}\right)^2 + m^2} + \alpha m, \qquad E^{s=-1} = \sqrt{\mathbf{p}^2\left(1 + \alpha\frac{m}{p}\right)^2 + m^2} + \alpha m \tag{13}$$

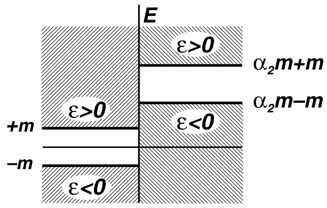

Figure 1. The interface between the vacuum (left-hand side of the picture) and the matter (right-hand side of the picture) with the corresponding neutrino band-gaps is shown. The parameter $\alpha = \alpha_2 > 2$.

that determine the positive- and negative-helicity eigenstates, respectively. The energies in (13) correspond to the particle (neutrino) solutions in the background matter. The two other values for the energy, corresponding to the negative sign $\varepsilon = -1$, are for the antiparticle solutions. As usual, by changing the sign of energy, we obtain the values

$$\tilde{E}^{s=+1} = \sqrt{\mathbf{p}^2\left(1 - \alpha\frac{m}{p}\right)^2 + m^2} - \alpha m, \qquad \tilde{E}^{s=-1} = \sqrt{\mathbf{p}^2\left(1 + \alpha\frac{m}{p}\right)^2 + m^2} - \alpha m \tag{14}$$

that correspond to the positive- and negative-helicity antineutrino states in the matter. The neutrino dispersion relations in matter exhibits a very fascinating feature (see also [15, 16]): the neutrino energy may have a minimum at nonzero momentum. It may also happen that the neutrino group and phase velocities are oppositely directed. The expressions in (13) and (14) would reproduce the neutrino dispersion relations of [16], if the contribution of the neutral-current interaction to the neutrino potential were omitted.

In the general case of matter composed of electrons, neutrons and protons the matter density parameter α for different neutrino species is

$$\alpha_{\nu_e,\nu_\mu,\nu_\tau} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}(n_e(4\sin^2\theta_W + \varrho) + n_p(1 - 4\sin^2\theta_W) - n_n), \tag{15}$$

where $\varrho = 1$ for the electron neutrino and $\varrho = -1$ for the muon and tau neutrinos.

Note that on the basis of the obtained energy spectrum (10) the neutrino trapping and reflection, the neutrino–antineutrino pair annihilation and creation in a medium can be studied [15, 16, 19–21].

2.3. Neutrino reflection, trapping and neutrino–antineutrino pair annihilation and creation in matter

Analysis of the obtained energy spectrum (13), (14) enables us to predict some interesting phenomena that may appear at the interface of the two media with different densities and, in particular, at the interface between matter and vacuum. Indeed, as it follows from (13) and (14) (see also [20]), the band-gap for neutrino and antineutrino in matter is displaced with respect to the vacuum case in neutrino mass and is determined by the condition $\alpha m - m \leqslant E < \alpha m + m$. For instance, if $\alpha = \alpha_2 > 2$ then there is no band-gap overlapping. This situation is illustrated in figure 1.

Let us consider first a neutrino moving in the vacuum towards the interface with energy that falls into the band-gap region in matter. In this case, the neutrino has no chance to survive in the matter and thus it is reflected from the interface. The same situation is realized for the antineutrino moving in the matter with energy falling into the band-gap in the vacuum. In this case, the antineutrino is trapped by the matter. When the energies of neutrino in the vacuum or antineutrino in the medium fall into the region between the two band-gaps the effects of the neutrino–antineutrino annihilation or pair creation may occur (see the first paper of [16, 19–21]). Indeed, the 'negative sign' energy levels in the matter (the right-hand side of figure 1) have their counterparts in the 'positive sign' energy levels in the vacuum (the left-hand side of figure 1). The neutrino–antineutrino pair creation can be interpreted as a process a particle state appearance in the 'positive sign' energy range accompanied by the appearance of the hole state in the 'negative sign' energy sea. The phenomenon of neutrino–antineutrino pair creation in the presence of matter is similar to the spontaneous electron–positron pair creation the electrodynamics according (Klein's paradox).

2.4. Neutrino quantum states in rotating medium

In this section, we apply our method to a particular case when a neutrino is propagating in a rotating medium of constant density [22]. Suppose that a neutrino is propagating perpendicular to uniformly rotating matter composed of neutrons. This can be considered for modelling of neutrino propagation inside a rotating neutron star. The corresponding modified Dirac equation for the neutrino wavefunction is given by (7) with the matter potential accounting for rotation,

$$f^{\mu} = -G(n, n\mathbf{v}), \qquad \mathbf{v} = (\omega y, 0, 0), \tag{16}$$

where $G = \frac{G_F}{\sqrt{2}}$. Here ω is the angular frequency of matter rotation around the OZ axis, it also is supposed that the neutrino propagates along the OY axis. For the neutrino wavefunction components $\Psi(x)$ we get the from the modified Dirac equation (7) a set of equations[1],

$$\begin{aligned}
&[\mathrm{i}(\partial_0 - \partial_3) + Gn]\Psi_1 + [-(\mathrm{i}\partial_1 + \partial_2) + Gn\omega y]\Psi_2 = m\Psi_3,\\
&[(-\mathrm{i}\partial_1 + \partial_2) + Gn\omega y]\Psi_1 + [\mathrm{i}(\partial_0 + \partial_3) + Gn]\Psi_2 = m\Psi_4,\\
&\mathrm{i}(\partial_0 + \partial_3)\Psi_3 + (\mathrm{i}\partial_1 + \partial_2)\Psi_4 = m\Psi_1,\\
&(\mathrm{i}\partial_1 - \partial_2)\Psi_3 + \mathrm{i}(\partial_0 - \partial_3)\Psi_4 = m\Psi_2.
\end{aligned} \tag{17}$$

In general case, it is not a trivial task to find solutions of this set of equations.

The problem is reasonably simplified in the limit of a very small neutrino mass, i.e. when the neutrino mass can be ignored in the left-hand side of (17) with respect to the kinetic and interaction terms in the right-hand sides of these equations. In this case, two pairs of the neutrino wavefunction components decouple one from each other and four equations (17) disintegrate to the two independent sets of two equations, that couple together the neutrino wavefunction components in pairs, (Ψ_1, Ψ_2) and (Ψ_3, Ψ_4).

The second pair of equations (17) does not contain a matter term and is attributed to the sterile right-handed chiral neutrino state, Ψ_R. The corresponding solution can be taken in the plain-wave form

$$\Psi_R \sim L^{-\frac{3}{2}} \exp\{\mathrm{i}(-p_0 t + p_1 x + p_2 y + p_3 z)\}\psi, \tag{18}$$

where p_μ is the neutrino momentum. Then for the components Ψ_3 and Ψ_4 we obtain from (17) the following equations:

$$\begin{aligned}
&(p_0 - p_3)\Psi_3 - (p_1 - \mathrm{i}p_2)\Psi_4 = 0,\\
&-(p_1 + \mathrm{i}p_2)\Psi_3 + (p_0 + p_3)\Psi_4 = 0.
\end{aligned} \tag{19}$$

[1] The chiral representation for Dirac matrixes is used.

Finally, from (19) for the sterile right-handed neutrino we get

$$\Psi_R = \frac{e^{-ipx}}{L^{3/2}\sqrt{2p_0(p_0 - p_3)}} \begin{pmatrix} 0 \\ 0 \\ -p_1 + ip_2 \\ p_3 - p_0 \end{pmatrix}, \tag{20}$$

where $px = p_\mu x^\mu$, $p_\mu = (p_0, p_1, p_2, p_3)$ and $x_\mu = (t, x, y, z)$. This solution, as it should be, has the vacuum dispersion relation.

In the neutrino mass vanishing limit the first pair of equations (17) corresponds to the active left-handed neutrino. The form of these equations is similar to the correspondent equations for a charged particle (e.g., an electron) moving in a constant magnetic field B given by the potential $\boldsymbol{A} = (By, 0, 0)$ (see, for instance, [9]). To display the analogy, we note that in our case the matter current components $n\boldsymbol{v}$ play the role of the vector potential $\boldsymbol{A}$. The existed analogy between an electron dynamics in an external electromagnetic field and a neutrino dynamics in background matter is further discussed in the conclusion (section 6).

The solution of the first pair of equations (17) can be taken in the form

$$\Psi_L \sim \frac{1}{L}\exp\{i(-p_0 t + p_1 x + p_3 z)\}\psi(y), \tag{21}$$

and for the components Ψ_1 and Ψ_2 of the neutrino wavefunction we obtain from (17) the following equations:

$$\begin{aligned} &(p_0 + p_3 + Gn)\Psi_1 - \sqrt{\rho}\left(\frac{\partial}{\partial\eta} - \eta\right)\Psi_2 = 0, \\ &\sqrt{\rho}\left(\frac{\partial}{\partial\eta} + \eta\right)\Psi_1 + (p_0 - p_3 + Gn)\Psi_2 = 0, \end{aligned} \tag{22}$$

where

$$\eta = \sqrt{\rho}\left(x_2 + \frac{p_1}{\rho}\right), \qquad \rho = Gn\omega. \tag{23}$$

For the wavefunction we finally get

$$\Psi_L = \frac{\rho^{\frac{1}{4}} e^{-ip_0 t + ip_1 x + ip_3 z}}{L\sqrt{(p_0 - p_3 + Gn)^2 + 2\rho N}} \begin{pmatrix} (p_0 - p_3 + Gn)\, u_N(\eta) \\ -\sqrt{2\rho N} u_{N-1}(\eta) \\ 0 \\ 0 \end{pmatrix}, \tag{24}$$

where $u_N(\eta)$ are Hermite functions of order N. For the energy of the active left-handed neutrino we get

$$p_0 = \sqrt{p_3^2 + 2\rho N} - Gn, \qquad N = 0, 1, 2, \ldots. \tag{25}$$

The energy depends on the neutrino momentum component p_3 along the rotation axis of matter and the quantum number N that determines the magnitude of the neutrino momentum in the orthogonal plane. For description of antineutrinos one has to consider the 'negative sign' energy eigenvalues (see similar discussion in section 2.2). Thus, the energy of an electron antineutrino in the rotating matter composed of neutrons is given by

$$\tilde{p}_0 = \sqrt{p_3^2 + 2\rho N} + Gn, \qquad N = 0, 1, 2, \ldots. \tag{26}$$

Obviously, generalization for different other neutrino flavours and matter composition is just straightforward (see (6) and (15)).

Thus, it is shown [22] that the transversal motion of an active neutrino and antineutrino is quantized in moving matter very much like an electron energy is quantized in a constant magnetic field that corresponds to the relativistic form of the Landau energy levels (see, for instance, the first book of [9]). Consider again antineutrino. The transversal motion of momentum is given by

$$\tilde{p}_{\perp} = \sqrt{2\rho N}. \tag{27}$$

The quantum number N determines also the radius of the antineutrino quasi-classical orbit in matter (it is supposed that $N \gg 1$ and $p_3 = 0$),

$$R = \sqrt{\frac{2N}{Gn\omega}}. \tag{28}$$

It follows that antineutrinos can have bound orbits inside a rotating star. To make an estimation of magnitudes, let us consider a model of a rotating neutron star with radius $R_{\mathrm{NS}} = 10\,\mathrm{km}$, matter density $n = 10^{37}\,\mathrm{cm}^{-3}$ and angular frequency $\omega = 2\pi \times 10^3\,\mathrm{s}^{-1}$. For this set of parameters, the radius of an antineutrino orbits is less than the typical star radius R_{NS} if the quantum number $N \leqslant N_{\max} = 10^{10}$. Therefore, antineutrinos that occupy orbits with $N \leqslant 10^{10}$ can be bounded inside the star. The scale of the bounded antineutrinos energy estimated by (26) is of the order $\tilde{p}_0 \sim 1\,\mathrm{eV}$. It should be underlined that within the quasi-classical approach the neutrino binding on circular orbits is due to an effective force that is orthogonal to the particle speed. Note that there is another mechanism of neutrinos binding inside a neutron star when the effect is produced by a gradient of the matter density [19] (see also the conclusion).

2.5. Modified Dirac–Pauli equation for neutrino in matter

To derive the quantum equation for a neutrino wavefunction in the background matter we start with the well-known Dirac–Pauli equation for a neutral fermion with nonzero magnetic moment. For a massive neutrino moving in an electromagnetic field $F_{\mu\nu}$ this equation is given by

$$\left(\mathrm{i}\gamma^{\mu}\partial_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\Psi(x) = 0, \tag{29}$$

where m and μ are the neutrino mass and magnetic moment [23] [2], $\sigma^{\mu\nu} = \mathrm{i}/2(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu})$. It is worthwhile to be noted here that equation (29) can be obtained in the linear approximation over the electromagnetic field from the Dirac–Schwinger equation, which in the case of the neutrino takes the following form [25]:

$$(\mathrm{i}\gamma^{\mu}\partial_{\mu} - m)\Psi(x) = \int M^{F}(x', x)\Psi(x')\,\mathrm{d}x', \tag{30}$$

where $M^{F}(x', x)$ is the neutrino mass operator in the presence of the external electromagnetic field.

Recently in a series of our papers [26, 27] (see also [28]) we have developed the quasi-classical approach to the massive neutrino spin evolution in the presence of external fields and background matter. In particular, we have shown that the well-known Bargmann–Michel–Telegdi (BMT) equation [29] of the electrodynamics can be generalized for the case of a neutrino moving in the background matter and being also under the influence of external electromagnetic fields. The proposed new equation for a neutrino, which simultaneously

[2] For the recent studies of a massive neutrino electromagnetic properties, including discussion on the neutrino magnetic moment, see [24]

accounts for the electromagnetic interaction with external fields and also for the weak interaction with particles of the background matter, was obtained from the BMT equation by the following substitution of the electromagnetic field tensor $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$:

$$F_{\mu\nu} \to E_{\mu\nu} = F_{\mu\nu} + G_{\mu\nu}, \tag{31}$$

where the tensor $G_{\mu\nu} = (-\mathbf{P}, \mathbf{M})$ accounts for the neutrino interactions with particles of the environment. The substitution (31) implies that in the presence of matter the magnetic $\mathbf{B}$ and electric $\mathbf{E}$ fields are shifted by the vectors $\mathbf{M}$ and $\mathbf{P}$, respectively,

$$\mathbf{B} \to \mathbf{B} + \mathbf{M}, \qquad \mathbf{E} \to \mathbf{E} - \mathbf{P}. \tag{32}$$

We have also shown [26, 27] how to construct the tensor $G_{\mu\nu}$ with the use of the neutrino speed, matter speed and matter polarization 4-vectors.

Now let us consider the case of a neutrino moving in matter without any electromagnetic field in the background. Starting from the Dirac–Pauli equation (29) for a neutrino in electromagnetic field $F_{\mu\nu}$, we apply the substitution (31) which now becomes

$$F_{\mu\nu} \to G_{\mu\nu}. \tag{33}$$

As a result of this substitution, we obtain the quantum equation for the neutrino wavefunction in the presence of the background matter in the form [30]

$$\left(\mathrm{i}\gamma^{\mu}\partial_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}G_{\mu\nu}\right)\Psi(x) = 0 \tag{34}$$

that can be regarded as the modified Dirac–Pauli equation.

Consider an explicit solution of the obtained equation (34) for the case of an unpolarized matter composed of only electrons we have

$$G^{\mu\nu} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}\gamma n \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_3 & \beta_2 \\ 0 & \beta_3 & 0 & -\beta_1 \\ 0 & -\beta_2 & \beta_1 & 0 \end{pmatrix}, \qquad \gamma = (1-\beta^2)^{-1/2}, \tag{35}$$

where $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)$ is the neutrino three-dimensional speed and n denotes the number density of the background electrons. From (35) and two equations, (29) and (34), it is possible to see that the term $\frac{\tilde{G}_F}{2\sqrt{2}\mu}\gamma n\boldsymbol{\beta}$ in (34) plays the role of the magnetic field $\mathbf{B}$ in (29). The corresponding neutrino energy spectrum is

$$E = \sqrt{\mathbf{p}^2(1+\alpha^2) + m^2 - 2\alpha mps}, \qquad \alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m}. \tag{36}$$

This expression can be transformed to the form

$$E = \sqrt{\mathbf{p}^2 + m^2\left(1 - s\frac{\alpha p}{m}\right)^2} \tag{37}$$

that can be obtained from the neutrino vacuum spectrum by the formal shift of the neutrino mass $m \to m\left(1 - s\frac{\alpha p}{m}\right)$.

The exact solution of the Dirac–Pauli equation (34) can be obtained in the following form [30]:

$$\Psi_{\mathbf{p},s}(\mathbf{r},t) = \frac{\mathrm{e}^{-\mathrm{i}(Et-\mathbf{pr})}}{2L^{\frac{3}{2}}} \begin{pmatrix} \sqrt{1+\frac{m-s\alpha p}{E}}\sqrt{1+s\frac{p_3}{p}} \\ s\sqrt{1+\frac{m-s\alpha p}{E}}\sqrt{1-s\frac{p_3}{p}}\,\mathrm{e}^{\mathrm{i}\delta} \\ s\sqrt{1-\frac{m-s\alpha p}{E}}\sqrt{1+s\frac{p_3}{p}} \\ \sqrt{1-\frac{m-s\alpha p}{E}}\sqrt{1-s\frac{p_3}{p}}\,\mathrm{e}^{\mathrm{i}\delta} \end{pmatrix}. \tag{38}$$

In the limit of vanishing matter density, when $\alpha \to 0$, this wavefunction transforms to the vacuum solution of the Dirac equation.

The obtained neutrino energy spectrum (36), for not extremely high matter densities $\alpha \frac{pm}{E_0^2} \ll 1$, yields the correct result for the energy difference $\Delta E = E(s=-1) - E(s=+1)$ of the two neutrino helicity states

$$\Delta E \approx 2m\alpha \frac{p}{E_0}, \tag{39}$$

where we use the notation $E_0 = \sqrt{p^2+m^2}$. Therefore, on the basis of the obtained exact solution for the neutrino wavefunction in the case of relativistic neutrinos one can derive the probability of spin oscillations $\nu_L \leftrightarrow \nu_R$ in transversal magnetic field with the correct form of the matter term [31].

2.6. *Flavour and helicity neutrino energy difference in matter*

Although the neutrino energy spectra correspondent to the modified Dirac and Dirac–Pauli equations, (7) and (34), are not the same, an equal result given by (39) for the energy difference $\Delta E = E(s=-1) - E(s=+1)$ of the two neutrino helicity states can be obtained from both of the spectra in the low matter density or high-energy limit $\alpha \frac{pm}{E_0^2} \ll 1$.

It should be also noted that for the relativistic neutrinos the energy spectrum for the neutrino helicity states of equation (10) in the low-density limit reproduces the correct energy values for the neutrino left-handed and right-handed chiral states,

$$E_{\nu_L} \approx E(s=-1) \approx E_0 + \frac{\tilde{G}_F}{\sqrt{2}} n, \qquad E_{\nu_R} \approx E(s=-1) \approx E_0, \tag{40}$$

as it should be for the active left-handed and sterile right-handed neutrino in matter.

We should like to note, that the obtained spectra for the flavour neutrinos of different helicities in the presence of matter enables one to reproduce the well-known result for the energy difference of two flavour neutrinos in matter. In order to demonstrate this we expand the expressions for the relativistic electron and muon neutrino energies (given by (10) for the Dirac case or by (53) for the Majorana case), over $m/p \ll 1$ and get

$$E^{s=-1}_{\nu_e,\nu_\mu} \approx E_0 + 2\alpha_{\nu_e,\nu_\mu} m. \tag{41}$$

Then the energy difference for the two active flavour neutrinos is

$$\Delta E = E^{s=-1}_{\nu_e} - E^{s=-1}_{\nu_\mu} = \sqrt{2} G_F n_e. \tag{42}$$

Analogously, considering the spin-flavour oscillations $\nu_{e_L} \rightleftarrows \nu_{\mu_R}$, for the corresponding energy difference we find

$$\Delta E = E^{s=-1}_{\nu_e} - E^{s=+1}_{\nu_\mu} = \sqrt{2} G_F \left(n_e - \tfrac{1}{2} n_n\right). \tag{43}$$

These equations enable one to get the expressions for the neutrino flavour and spin-flavour oscillation probabilities with resonance dependence on matter density in the complete agreement with the results of [31, 32].

2.7. *Modified Dirac–Pauli equation in magnetized and polarized matter*

It is also possible to generalize the Dirac–Pauli equation (29) (or (34)) for the case when a neutrino is moving in a magnetized background matter. For this case (i.e., when the effects of matter and magnetic field on neutrino have to be accounted for simultaneously) the modified Dirac–Pauli equation is [30]

$$\left\{ \mathrm{i}\gamma^\mu \partial_\mu - m - \frac{\mu}{2} \sigma^{\mu\nu} (F_{\mu\nu} + G_{\mu\nu}) \right\} \Psi(x) = 0. \tag{44}$$

The neutrino energy in the magnetized matter can be obtained from (36) by the following redefinition:

$$\alpha \to \alpha' = \alpha + \frac{\mu B_\parallel}{p}, \tag{45}$$

where $B_\parallel = (\mathbf{B}\mathbf{p})/p$ is the longitudinal to the neutrino momentum magnetic field component. Thus, the neutrino energy in this case reads

$$E = \sqrt{\mathbf{p}^2 + m^2\left(1 - s\frac{\alpha p + \mu B_\parallel}{m}\right)^2}. \tag{46}$$

For the relativistic neutrinos the expression of equation (37) gives, in the linear approximation over the matter density and the magnetic field strength, the correct value (see [26–28]) for the energy difference of the two opposite helicity states in the magnetized matter,

$$\Delta_{\text{eff}} = \frac{\tilde{G}_F}{\sqrt{2}}n + 2\frac{\mu B_\parallel}{\gamma}. \tag{47}$$

Note that the problem of the neutrino dispersion relation in an external magnetic field and matter was also studied previously in many papers with use of different methods [34].

Now we can consider the neutrino spin oscillations in the presence of non-moving matter being under the influence of an arbitrary constant magnetic field $\mathbf{B} = \mathbf{B}_\parallel + \mathbf{B}_\perp$, here $\mathbf{B}_\perp$ is the transversal to the neutrino momentum component of the external field. In the adiabatic approximation the probability of the oscillations $\nu_L \leftrightarrow \nu_R$ can be written in the form

$$P_{\nu_L\to\nu_R}(x) = \sin^2 2\theta_{\text{eff}} \sin^2\frac{\pi x}{L_{\text{eff}}}, \qquad \sin^2 2\theta_{\text{eff}} = \frac{E_{\text{eff}}^2}{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}, \qquad L_{\text{eff}} = \frac{2\pi}{\sqrt{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}}, \tag{48}$$

where $E_{\text{eff}} = 2\mu B_\perp$ (terms $\sim \gamma^{-1}$ are omitted here), and x is the distance travelled by the neutrino.

Let us now shortly discuss the effect of matter polarization. Consider the case of matter composed of electrons in the presence of such strong background magnetic field so that the following condition is valid $B > \frac{p_F^2}{2e}$, where $p_F = \sqrt{\mu^2 - m_e^2}$, μ and m_e are, respectively, the Fermi momentum, chemical potential and mass of electrons. Then all of the electrons occupy the lowest Landau level; therefore the matter is completely polarized in the direction opposite to the unit vector $\frac{\mathbf{B}}{B}$. From the general expression for the tensor $G_{\mu\nu}$ (see the second paper of [26]) we get [30]

$$G^{\mu\nu} = \frac{1}{2\sqrt{2}\mu}\gamma n\left\{\tilde{G}_F\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & -\beta_3 & \beta_2\\ 0 & \beta_3 & 0 & -\beta_1\\ 0 & -\beta_2 & \beta_1 & 0\end{pmatrix} - G_F\begin{pmatrix}0 & -\beta_2 & \beta_1 & 0\\ \beta_2 & 0 & -\beta_0 & 0\\ -\beta_1 & \beta_0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\right\}. \tag{49}$$

Thus, the modified Dirac–Pauli equation (44) with the tensor $G_{\mu\nu}$ given by (49) can be used for description of the neutrino motion in matter which is magnetized and totally polarized in respect to the magnetic field vector $\boldsymbol{B}$ direction. The neutrino energy in such a case can be obtained from (36) by the following redefinition:

$$\alpha \to \tilde{\alpha} = \alpha\left[1 - \frac{\text{sign}\left(\frac{B_\parallel}{B}\right)}{1 + \sin^2 4\theta_W}\right] + \frac{\mu B_\parallel}{p}. \tag{50}$$

In equation (50), the second term in brackets accounts for the effect of the matter polarization. It follows, that the effect of the matter polarization can reasonably change the total matter contribution to the neutrino energy (46) (see also [33]).

Note that the problem of the neutrino dispersion relation in an external magnetic field and matter was previously also studied in many papers with use of different methods [34].

2.8. Majorana neutrino

We have considered so far the case of the Dirac neutrino. Now let us turn to the Majorana neutrino [20]. For a Majorana neutrino we derive the following contribution to the effective Lagrangian accounting for the interaction with the background medium,

$$\Delta L_{\text{eff}} = -f^{\mu}(\bar{\nu}\gamma_{\mu}\gamma^{5}\nu), \tag{51}$$

which leads to the Dirac equation

$$\{i\gamma_{\mu}\partial^{\mu} - \gamma_{\mu}\gamma_{5}f^{\mu} - m\}\Psi(x) = 0. \tag{52}$$

This equation differs from the one, obtained in the Dirac case, by doubling of the interaction term and lack of the vector part. The corresponding energy spectrum for equation (52) is

$$E_{\varepsilon} = \varepsilon\sqrt{\mathbf{p}^{2}\left(1 - 2s\alpha\frac{m}{p}\right)^{2} + m^{2}}. \tag{53}$$

From this expression it is clear that the energy of the Majorana neutrino has its minimal value equal to the neutrino mass, $E = m$. This means that no effects are anticipated for the Majorana neutrino such as the Dirac neutrino has at the two media interface and which are discussed above. So that, in particular, there is no Majorana neutrino trapping and reflection by matter. It should be noted that equation (52) and the Majorana neutrino spectrum in matter were discussed previously also in [16, 35].

3. Neutrino Green function in matter

The neutrino Green function, along with the wavefunction, is an important characteristic of the neutrino (propagation) in matter. Developing further the method of the exact solutions for the studies of the neutrino propagation in matter, we consider explicit Green functions for the modified Dirac equation for the Dirac and Majorana neutrinos [36]. For the Dirac and Majorana neutrino Green functions we obtain the same equations as for the correspondent wavefunctions, (7) and (52), with the only difference that $-\delta(x)$ functions stay on the right-hand sides. In the momentum representation the equation for the Green function has the following form:

$$\left\{i\gamma_{\mu}\partial^{\mu} - \tfrac{1}{2}\gamma_{\mu}(a+\gamma_{5})f^{\mu} - m\right\}G(p) = -1, \tag{54}$$

where $a = 1$ for the Dirac neutrino and $a = 0$ for the Majorana case. Squaring the left-hand side operator, it is possible to obtain the following expression for the Green function of neutrino in matter:

$$G(q) = \frac{-(q^{2}-m^{2})(\hat{q}+m) + \hat{f}(\hat{q}-m)P_{L}(\hat{q}+m) - f^{2}\hat{q}P_{L} + 2(fq)P_{R}(\hat{q}+m)}{(q^{2}-m^{2})^{2} - 2(fq)(q^{2}-m^{2}) + f^{2}q^{2}}, \tag{55}$$

where

$$\begin{aligned} &q = p - \tfrac{1}{2}(a-1)f, \qquad \hat{q} = q_{\mu}\gamma^{\mu}, \qquad q^{2} = q_{\mu}q^{\mu}, \\ &\hat{f} = f_{\mu}\gamma^{\mu}, \qquad (fq) = f_{\mu}q^{\mu}, \qquad P_{L,R} = \tfrac{1}{2}(1\pm\gamma_{5}). \end{aligned} \tag{56}$$

Now let us consider the denominator of expression (55). The poles of the Green function determine the neutrino dispersion relation. Equating the denominator to zero, we obtain quadratic equation relative to q_0,

$$(q^2 - m^2)^2 - 2(fq)(q^2 - m^2) + f^2 q^2 = 0. \tag{57}$$

In some special cases equation (57) can be solved analytically. One of such cases is that of uniform medium, moving at constant speed $\mathbf{v}$ parallel to the neutrino momentum $\mathbf{p}$. In this case, we can solve equation (57) for q_0 and then find p_0,

$$p_0 = \frac{1}{2}\left[af_0 + s|\mathbf{f}| + \epsilon\sqrt{4m^2 + \left(2|\mathbf{p} - \frac{1}{2}(a-1)\mathbf{f}| - s(f_0 + s|\mathbf{f}|)\right)^2}\right]. \tag{58}$$

There are four solutions of (58) correspondent to $s = \pm 1$ and $\epsilon = \pm 1$. From equation (58) one can find that all solutions except one are of definite sign for any $|\mathbf{p}|$. The sign of p_0 for $\epsilon = -1$ and $s = 1$ however can be both positive and negative for different $|\mathbf{p}|$. One can also note that in case of

$$af_0 + |\mathbf{f}| < 2m, \tag{59}$$

the sign of this p_0 is always negative.

In case condition equation (59) holds, the solution of equations (7) and (52) can be expressed in the form of the superposition of plane waves each with a definite sign of energy. Note that if condition (59) is violated then there exists a plane wave that has positive energy for some $|\mathbf{p}|$ and negative for others. Stated in other words, condition (59) means that Green function (55) can be chosen causal by imposing special rules of poles bypassing (negative poles should be bypassed from below and positive poles should be bypassed from above). Once we have got the causal Green function the perturbation technique can be developed for the description of the neutrino propagation in matter.

Another way to interpret condition (59) is to turn attention to [20] where it was shown, that for the matter at rest, the spontaneous $\nu\tilde{\nu}$ pair creation can take place only when $f_0 > 2m$. From the analysis of the allowed energy zones for neutrino in matter it follows that $\nu\tilde{\nu}$ pair creation in moving matter can take place only when $af_0 + |\mathbf{f}| > 2m$. So that the possibility of using the neutrino Green function (55) is limited by the particular value of matter density when $\nu\tilde{\nu}$ pair creation processes become available.

For the Majorana neutrino moving through uniform matter at rest, condition (59) is always valid for any matter densities f_0 because $a = 0$ and $\mathbf{f} = 0$ in this case.

4. Electron wavefunction and energy spectrum in matter

In [3, 5–7], it has been shown how the approach, developed at first for description of a neutrino motion in the background matter, can be spread for the case of an electron propagating in matter.

Let us consider an electron having the standard model interactions with particles of electrically neutral matter composed of neutrons, electrons and protons. This can be used for modelling a real situation which existed, for instance, when electrons move in different astrophysical environments. We suppose that there is a macroscopic amount of the background particles in the scale of an electron de Broglie wavelength. To further simplify the model, we consider the case of nuclear matter [21, 37] composed of neutrons. Then the addition to the electron effective interaction Lagrangian is

$$\Delta L_{\text{eff}}^{(e)} = -f^{\mu}\left(\bar{e}\gamma_{\mu}\frac{1 - 4\sin^2\theta_W + \gamma^5}{2}e\right), \tag{60}$$

where the explicit form of f^{μ} depends on the background particles density, speed and polarization and is determined by (4) and (5). The modified Dirac equation for the electron wavefunction in matter is [3]

$$\left\{ i\gamma_{\mu}\partial^{\mu} - \tfrac{1}{2}\gamma_{\mu}(1 - 4\sin^2\theta_W + \gamma_5)\widetilde{f}^{\mu} - m_e \right\}\Psi_e(x) = 0, \tag{61}$$

where for the case of an electron moving in the background of neutrons

$$\tilde{f}^{\mu} = -f^{\mu} = \frac{G_F}{\sqrt{2}}\left(j_n^{\mu} - \lambda_n^{\mu}\right). \tag{62}$$

We consider below unpolarized neutrons so that

$$\widetilde{f}^{\mu} = \frac{G_F}{\sqrt{2}}(n_n, n_n\mathbf{v}), \tag{63}$$

here n_n is the neutrons number density and $\mathbf{v}$ is the speed of the reference frame in which the mean momentum of the neutrons is zero. The corresponding electron energy spectrum in the case of unpolarized matter at rest is given by

$$E_{\varepsilon}^{(e)} = \varepsilon\eta_e\sqrt{\mathbf{p}^2\left(1 - s_e\alpha_n\frac{m_e}{p}\right)^2 + {m_e}^2} + c\alpha_n m_e, \qquad \alpha_n = \frac{1}{2\sqrt{2}}G_F\frac{n_n}{m_e}, \tag{64}$$

where $c = 1 - 4\sin^2\theta_W$, $\eta_e = \mathrm{sign}\left(1 - s\alpha_n\frac{m_e}{p}\right)$, m_e and p are the electron mass and momentum.

For the wavefunction of the electron moving in nuclear matter we get [5–7]

$$\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t) = \frac{e^{-i(E_{\varepsilon}^{(e)}t - \mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix} \sqrt{1 + \frac{m_e}{E_{\varepsilon}^{(e)} - c\alpha_n m_e}}\sqrt{1 + s\frac{p_3}{p}} \\ s\sqrt{1 + \frac{m_e}{E_{\varepsilon}^{(e)} - c\alpha_n m_e}}\sqrt{1 - s\frac{p_3}{p}}\,e^{i\delta} \\ s\varepsilon\eta_e\sqrt{1 - \frac{m_e}{E_{\varepsilon}^{(e)} - c\alpha_n m_e}}\sqrt{1 + s\frac{p_3}{p}} \\ \varepsilon\eta_e\sqrt{1 - \frac{m_e}{E_{\varepsilon}^{(e)} - c\alpha_n m_e}}\sqrt{1 - s\frac{p_3}{p}}\,e^{i\delta} \end{pmatrix}. \tag{65}$$

The exact solutions of this equation open a new method for investigation of different quantum processes which can appear when electrons propagate in matter.

5. Neutrino and electron spin light in matter

In this section, we illustrate how the developed method based on the use of the exact solutions of the modified Dirac equations for neutrino and electron wavefunctions can be used in the study of different phenomena that arise when a neutrino or electron move in matter. As an example, we discuss below the spin light of neutrino (SLν) and the spin light of electron (SLe), new types of electromagnetic radiation that can be produced by the Dirac particles while moving in the background matter. The spin light of neutrino in matter, one of the four new phenomena studied in our recent papers (see for a short review [38]), is an electromagnetic radiation that can be emitted by a massive neutrino (due to its nonzero magnetic moment) when the particle moves in the background matter. Within the quasi-classical treatment the existence of the SLν was first proposed and studied in [39], while the quantum theory of this phenomenon was developed in [1–3, 20, 30, 28, 40]. The spin light of electron in matter [3, 5–7] also originates from the particle magnetic moment procession in matter. Note that the term 'spin light of electron' was used first in [41] for designation of the particular spin-dependent contribution to the electron synchrotron radiation power. It should be stressed that SLν and SLe in matter are really new mechanisms of electromagnetic radiation of quite a

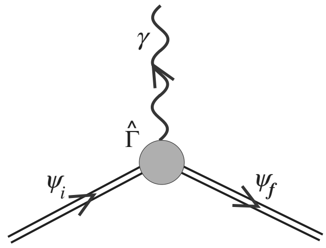

Figure 2. The SLν and SLe radiation diagram.

different nature than ones considered before including the Cherenkov radiation of particles in medium. In particular, the spin light processes may proceed even when the photon refractive index in matter equals to $n_\gamma = 1$.

The corresponding Feynman diagram of these processes is shown in figure 2.
The particles initial ψ_i and final ψ_f states (shown by 'broad lines') are exact solutions of the corresponding Dirac equations for the neutrino and electron in matter that account for the particles interaction with matter. The amplitude of the SLν process is given by

$$S_{fi} = -\mu\sqrt{4\pi}\int \mathrm{d}^4x\bar{\psi}_f(x)(\hat{\Gamma}\,\mathbf{e}^*)\frac{\mathrm{e}^{\mathrm{i}kx}}{\sqrt{2\omega L^3}}\psi_i(x), \qquad \hat{\Gamma} = \mathrm{i}\omega\{[\,\Sigma\times\varkappa\,] + \mathrm{i}\gamma^5\,\Sigma\}, \tag{66}$$

where μ is the neutrino magnetic moment, $k^\mu = (\omega, \mathbf{k})$ and $\mathbf{e}^*$ are the photon momentum and polarization vectors, $\varkappa = \mathbf{k}/\omega$ is the unit vector pointing in the possible direction of the emitted photon propagation. The amplitude of the process SLe is given by

$$S_{fi} = -\mathrm{i}e\sqrt{4\pi}\int \mathrm{d}^4x\bar{\psi}_f(x)(\gamma^\mu e^*_\mu)\frac{\mathrm{e}^{\mathrm{i}kx}}{\sqrt{2\omega L^3}}\psi_i(x), \tag{67}$$

where $-e$ is the electron charge. The further evaluation of the SLν and SLe characteristics of the processes, such as the differential and total rates and powers, angular distributions etc, can be found in the above-mentioned papers. From the energy–momentum conservation in the SLν and SLe processes we obtain the following values for the spin light radiation energy:

$$\omega_{\mathrm{SL}} = \frac{2\alpha_n m_l p[\tilde{E} - (p + \alpha_n m_l)\cos\theta_{\mathrm{SL}}]}{(\tilde{E} - p\cos\theta_{\mathrm{SL}})^2 - (\alpha_n m_l)^2}, \qquad \tilde{E} = E - c\alpha_n m_l, \qquad \alpha_n = \frac{1}{2\sqrt{2}}G_F\frac{n_n}{m_l}, \tag{68}$$

where θ_{SL} is the angle between possible directions of the radiation and the initial particle momentum $\mathbf{p}$, for the case of neutrinos $m_l = m_\nu$ and $c_l = c_\nu = 1$, whereas for electrons $m_l = m_e$ and $c_l = c_e = 1 - 4\sin^2\theta_W$. From (68) it follows that for the relativistic particles and a wide range of matter densities (that can be found in diverse astrophysical and cosmological environments) the energy range of SLν and SLe may even extend up to energies peculiar to the spectrum of gamma rays (see also [1, 3]).

For the rate of SLν in the case of ultra-relativistic neutrinos ($p \gg m$) we obtained [1, 2]

$$\Gamma_{\mathrm{SL}\nu} = 4\mu^2\alpha^2 m_\nu^2 p, \qquad m_\nu/p \ll \alpha \ll p/m_\nu, \tag{69}$$

where the matter density parameter α is given by (15); in case of negative α, SLν can be emitted by antineutrino. The main properties of SLν investigated in [1, 2, 39] can be summarized as follows [42]: (1) a neutrino with nonzero mass and magnetic moment when moving in dense matter can emit spin light; (2) in general, SLν in matter is due to the dependence of the neutrino dispersion relation in matter on the neutrino helicity; (3) the SLν radiation rate and power depend on the neutrino magnetic moment and energy, and also on the matter density; (4) the matter density parameter α, that depends on the type of neutrino and matter composition, can be negative; therefore, the types of initial and final neutrino (and antineutrino) states, conversion between which can effectively produce the SLν radiation, are determined by the matter composition; (5) SLν in matter leads to the neutrino-spin polarization effect; depending on the type of the initial neutrino (or antineutrino) and matter composition the negative-helicity relativistic neutrino (the left-handed neutrino ν_L) is converted to the positive-helicity neutrino (the right-handed neutrino ν_R) or vice versa; (6) the obtained expressions for the SLν radiation rate and power exhibit non-trivial dependence on the density of matter and on the initial neutrino energy; the SLν radiation rate and power are proportional to the neutrino magnetic moment squared which is, in general, a small value and also on the neutrino energy, that is why the radiation discussed can be effectively produced only in the case of ultra-relativistic neutrinos; (7) for a wide range of matter densities the radiation is beamed along the neutrino momentum, however the actual shape of the radiation spatial distribution may vary from projector-like to cap-like, depending on the neutrino momentum-to-mass ratio and the matter density; (8) in a wide range of matter densities the SLν radiation is characterized by total circular polarization; (9) the emitted photon energy is also essentially dependent on the neutrino energy and matter density; in particular, in the most interesting for possible astrophysical and cosmology applications case of ultra-high-energy neutrinos, the average energy of the SLν photons is one third of the neutrino momentum. Considering the listed above properties of SLν in matter, we argue that this radiation can be produced by high-energy neutrinos propagating in different astrophysical and cosmological environments.

A remark on the possibility for Majorana neutrino to emit the spin light in matter should be made. Obviously, due to the absence of the magnetic moment, such radiation is not expected in this case. However, considering the transition between two neutrinos of different flavour, it is possible to produce an analogous effect via the transition magnetic moment, that Majorana neutrinos can posses.

Performing the detailed study of SLe in neutron matter [43] we have found for the total rate

$$\Gamma_{\mathrm{SL}e} = e^2 m_e^2 \big/ (2p)[\ln(4\alpha_n p/m_e) - 3/2], \qquad m_e/p \ll \alpha_n \ll p/m_e, \qquad (70)$$

where it is supposed that $\ln \frac{4\alpha_n p}{m_e} \gg 1$. It was also found that the relativistic electrons can loose nearly the whole of its initial energy due to the SLe mechanism.

It should be noted that discussion on possible impact of the background plasma on the SLν radiation mechanism have been started in [2]. Then effects of plasma for SLν and SLe were considered by other authors in [44]. These authors, after we explained in [42] that their first conclusion that in the presence of matter the process '$\nu_L \to \nu_R + \gamma^*$ is kinematically forbidden' was wrong, obtained the SLν rate with account for the photon dispersion in plasma. In the case of ultra-high-energy neutrino (i.e., in the only case when the time scale of the process can be much less than the age of the universe) the SLν rate of [44] exactly reproduces our result (69) obtained in [1, 2]. The final result for the SLe total rate in the second paper of [44] in the leading logarithmic term confirms our result (70) obtained in [43].

6. Conclusion

We have considered a framework for treating different interactions of neutrinos and electrons in the presence of matter. The method developed is based on use of exact solutions of modified Dirac equations that include correspondent matter potentials. It has been demonstrated how this method works in consideration of different quantum processes that can proceed in presence of matter.

Finally, let us consider the established, in sections 2.4 and 2.5, analogy between particles dynamics in the presence of electromagnetic fields and dynamics in matter. The developed semiclassical approach to description of the matter effect, driven by (electro)weak forces, is valid as long as interactions of particles with the background is coherent. This condition is satisfied when a macroscopic amount of the background particles are confined within the scale of a neutrino or electron de Broglie wavelength. For the relativistic neutrinos or electrons the following condition should be satisfied $\frac{n}{\gamma_l m_l^3} \gg 1$, where n is the number density of matter, $\gamma_l = \frac{E_l}{m_l}$ and ($l = \nu$ or e). In case of varying density of the background matter, there is an additional condition for applicability of the approach developed (see, for instance, [19, 27, 45]). The variation scale of matter density should be much larger than the de Broglie wavelength, $\left|\frac{\nabla n}{np}\right| \ll 1$.

We can further develop the established in section 2.5 analogy between a neutrino motion in a rotating matter and an electron motion in a magnetic field. It is possible to explain the neutrino quasiclassical circular orbits as a result of action of the attractive central force

$$\mathbf{F}_m^{(\nu)} = q_m^{(\nu)} \boldsymbol{\beta} \times \mathbf{B}_m, \qquad \mathbf{B}_m = \boldsymbol{\nabla} \times \mathbf{A}_m, \qquad \mathbf{A}_m = n\mathbf{v}, \tag{71}$$

where the effective neutrino 'charge' in matter (composed of neutrons in the discussed case) is $q_m^{(\nu)} = -G$, whereas $\mathbf{B}_m$ and $\mathbf{A}_m$ play the roles of effective 'magnetic' field and the correspondent 'vector potential'. Like the magnetic part of the Lorentz force, $\mathbf{F}_m^{(\nu)}$ is orthogonal to the neutrino speed $\boldsymbol{\beta}$.

It is possible to generalize the above-discussed description of the matter effect on neutrinos for the case when the matter density n is not constant. For the most general case the 'matter-induced Lorentz force' is given by

$$\mathbf{F}_m^{(\nu)} = q_m^{(\nu)} \mathbf{E}_m + q_m^{(\nu)} \boldsymbol{\beta} \times \mathbf{B}_m, \tag{72}$$

where the effective 'electric' and 'magnetic' fields are, respectively,

$$\mathbf{E}_m = -\boldsymbol{\nabla} n - \mathbf{v}\frac{\partial n}{\partial t} - n\frac{\partial \mathbf{v}}{\partial t} \tag{73}$$

and

$$\mathbf{B}_m = n\boldsymbol{\nabla} \times \mathbf{v} - \mathbf{v} \times \boldsymbol{\nabla} n. \tag{74}$$

Using (4) and (5) (see also (15)) these expressions can be generalized for a background composed of different matter species. The force acting on a neutrino, produced by the first term of the effective 'electric' field in the neutron matter, was considered in [19]. Note that the same quasiclassical treatment of a neutrino motion in the electron plasma was considered in [45].

To conclude, we argue that it is also possible to introduce the 'matter-induced Lorentz force' for an electron moving in background matter. The weak forces acting on a neutrino and an electron in matter are identical. Therefore, similar to the case of neutrino, we can write for the force acting on an electron $\mathbf{F}_m^{(e)}$ in background matter

$$\mathbf{F}_m^{(e)} = q_m^{(e)} \mathbf{E}_m + q_m^{(e)} \boldsymbol{\beta} \times \mathbf{B}_m, \tag{75}$$

where appropriate magnitude for the effective electron 'charge' in matter $q_m^{(e)}$ should be used. As it follows from (73) and (75), an accelerating force acts on an electron when it moves in background matter with nonvanishing gradient of density. Using this observation, we should like to discuss a new mechanism of electromagnetic radiation by an electron moving in the neutrino background ($m = \nu$) with non-zero gradient of its density. This situation can be realized in different astrophysical and cosmology settings. For instance, this phenomenon can exist when an electron propagates in the radial direction from a compact star object inside a dense environment composed predominantly of neutrinos that also move in the radial direction after they are emitted from a central part of the star. In this case, that total power of the radiation (in the quasiclassical limit) is given by

$$I = \frac{2}{3} q_{\nu}^{(e)} \left[\frac{\mathbf{a}^2}{(1-\beta^2)^2} + \frac{(\mathbf{a}\boldsymbol{\beta})^2}{(1-\beta^2)^3} \right], \tag{76}$$

where β is the electron speed and $\mathbf{a}$ is the electron acceleration induced by the gradient of the neutrino background density. We expect that the proposed mechanism of the electromagnetic radiation can be important in other astrophysics settings like one that can be realised in neutron stars, gamma-ray bursts and black holes.

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